LEVEL 6 Calculus Huddle 2019

1) The value of $\lim\limits_{h \to 0} \frac{(h-2)^2-4}{h}$ is

a) 1 b) 2 c) -1 d) 4 e) none of these

2) The value of $\lim\limits_{x \to 1} \frac{|x|-x}{x-1}$ is

a) 2 b) $\frac{1}{2}$ c) -$\frac{1}{2}$ d) 0 e) none of these

3) A particle moves along a straight line with equation of motion $s = t^2 - 3t + 2$. The value of t at which it reverses direction is: ($t =$ time, $s =$ distance/position)

a) $\frac{1}{2}$ b) 0 c) $\frac{3}{2}$ d) $\frac{2}{3}$ e) none of these

4) At what value or values of x is the funcion below not differentiable?

$$f(x) = \begin{cases} x+2 \ \ if\, x \leq -1 \\ x^2 \ \ if \ -1 < x < 1 \\ 3-x \ \ if\, 1 \leq x \end{cases}$$

a) 1 b) 0, 1 c) 0, -1 d) 1, -1 e) none of these

5) The difference in height of the two horizontal tangents to the curve $y = x^3 + x^2 - x$ is:

a) $\frac{11}{9}$ b) $\frac{22}{27}$ c) $\frac{32}{27}$ d) $\frac{5}{3}$ e) none of these

6) There are two lines tangent to the curve $y = x^2 + 1$ that pass through the origin. One of the lines has a slope:

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) 1 d) 2 e) none of these

7) For how many values of x does the curve $y = x^3 + 2x$ have a tangent line parallel to the line $y = x$?

a) 1 b) 2 c) -3 d) 4 e) none of these

8) Let $f(x) = \frac{x}{sin\, x}$. The value of $f'(\pi/4)$ is

a) $1 + \pi/4$ b) $1 + \pi/2$ c) $\sqrt{2}\,(1 + \pi/2)$ d) $\sqrt{2}\,(1 - \pi/4)$ e) none of these

9) The value of $\lim\limits_{x \to 0^+} \frac{\sin 2x \cos 4x}{\tan 3x}$ is:

a) 0 b) $\frac{2}{3}$ c) $\frac{4}{3}$ d) $\frac{8}{3}$ e) none of these

10) An equation of the tangent line to the curve $y = \frac{8}{\sqrt{4+3x}}$ at the point (4, 2) is:

a) $6x + y = 26$ b) $3x - 4y = 4$ c) $4x + 15y = 46$ d) $3x + 16y = 44$ e) none of these

11) A definition of e is

a) $\lim_{n\to\infty}(1 - \frac{1}{n})^n$ b) $\lim_{n\to 0}(1 - n)^{1/n}$ c) $\lim_{n\to\infty}(n - \frac{1}{n})^n$ d) $\lim_{n\to\infty}(\frac{n^2+1}{n})^n$ e) none of these

12) If $x^2 - xy + y^3 = 8$, then an expression for $\frac{dy}{dx}$ is:

a) $\frac{y-2x}{3y^2-x}$ b) $\frac{-2y}{x+3y^2}$ c) $-\frac{3x^2+y}{2x-y}$ d) $\frac{x+y}{x^2+2y}$ e) none of these

13) A particle moves along a straight line with equation of motion $s = t^3 + t^2$ (s = distance, t = time). Find the value of t at which the accleration is equal to zero.

a) $-\frac{2}{3}$ b) $-\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{3}$ e) none of these

14) If $f(x) = g(g(x))$, find an expression for $f''(x)$ in terms of $g(x)$ and its derivatives.

a) $2g''(x)\,g(x) + 2(g'(x))^2$ b) $g''(g'(x))$ c) $g''(g(x))\,(g'(x))^2 + g'(g(x))\,g''(x)$
d) $g'(g'(x))g''(x) + (g'(x))^2$ d) $2\sqrt{2}$ e) none of these

15) A spherical balloon is being inflated such that the radius increases at a rate of one cm/s. In cm^3/s, how fast is the volume increasing three seconds after inflation begins? (Assume $r = 0$ when $t = 0$).

a) 36 b) 12π c) 24 d) 18π e) none of these

16) The length of a rectangle is decreasing at the rate of one foot per second, but the area remains constant. If the length is ten feet and the width is five feet, in feet per second, how fast is the width increasing?

a) $\frac{1}{2}$ b) 4 c) $\frac{1}{10}$ d) 2 e) none of these

17) Two cars are each 100 miles from Modesto. One is directly to the north and the other is directly to the east. The car to the north is heading toward Modesto at 60 mph, and the one to the east is heading toward Modesto at 30 mph. In miles per hour, how fast is the distance between the cars decreasing?

a) $200\sqrt{2}$ b) $100\sqrt{2}$ c) $50\sqrt{2}$ d) $45\sqrt{2}$ e) none of these

18) Let $y = x^2$, $x = 1$ and $\Delta x = 0.1$. Find the error $dy - \Delta y$ in approximating Δy by dy.

a) -.2 b) -.1 c) -.03 d) -.07 e) none of these

19) If Newton's method is used to find the cube root of a number "a" with a first approximation $x_1$with, the second approximation x_2 is

a) $x_2 = x_1 + \frac{3x_1^2}{x_1^3 - a}$ b) $x_2 = \sqrt[3]{x_1} - \sqrt[3]{a}$ c) $x_2 = x_1 - \frac{1}{3}x_1^{2/3}(x_1^{1/3} - a)$

d) $x_2 = x_1 - \frac{x_1^3 - a}{3x_1^2}$ e) none of these

20) The length of the largest interval for which the graph of $y = x^4 - 6x^3$ is concaved down is

a) $\frac{\sqrt{2}}{2}$ b) 3 c) $2\sqrt{2}$ d) 1 e) none of these

21) Let $f(x) = x^2$ on the interval [0, 2]. Let the interval be partitioned as follows. $P = \{0,\ 1.5,\ 2\}$. The value of the Riemann sum for this interval and partition by using the left endpoint is

a) $\frac{1}{2}$ b) $\frac{5}{16}$ c) $\frac{7}{16}$ d) $\frac{5}{8}$ e) none of these

22) Find the area of the region enclosed by $x^2 = 8y$ and $x - 2y + 8 = 0$

a) 24 units^2 b) 36 units^2 c) $78\frac{1}{3}$units^2 d) $79\frac{1}{3}$units^2 e) none of these

23) Does $f(x) = x^3 - 3x^2 - 3x + 1$ satisfy the hypotheses of the mean value theorem over the interval [0, 2]? If so, find all values of C in the interval [0, 2] that satisfy the conclusion of the therorem.

a) No b) Yes, $1+\sqrt{3}$ c) Yes, $\frac{3+\sqrt{3}}{3}$ d) Yes, $\frac{3\pm\sqrt{3}}{3}$ e) none of these

24) A ball is rolled across a level floor with an initial velocity of 28ft/sec. How many feet will the ball roll if the speed diminishes by 4 ft/sec^2 due to friction?

a) 26 b) 28 c) 49 d) 98 e) none of these

25) Let $f(x) = \int_{x^3}^{10} (t^2 + 1)^{60} dt$. Find the value of $f'(0)$.

a) 5 b) 1 c) 2 d) 0 e) none of these